Destruction of Fermion Zero Modes
on Cosmic Strings

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February 1, 2008

Abstract

I examine the existence of zero energy fermion solutions (zero modes) on cosmic strings in an $SO(10)$ grand unified theory. The current carrying capability of a cosmic string formed at one phase transition can be modified at subsequent phase transitions. I show that the zero modes may be destroyed and the conductivity of the string altered. I discuss the cosmological implications of this, and show that it allows vorton bounds to be relaxed.

1 Introduction

Cosmic strings, which are a type of topological defect, arise in many grand unified theories. Large quantities of them may be produced at phase transitions in the early universe. A network of cosmic strings could explain the observed anisotropy in the microwave background radiation and the large scale structure of the universe [1].

It has been realised in the past few years that cosmic strings have a far richer microstructure then previously thought [2]. In particular, the presence of conserved currents in the spectrum of a cosmic string has profound implications for the cosmology of the defects. In this paper I examine currents with fermion charge carriers, particularly massless ones.

Currents may provide a method of detecting strings. If they form at high energy scales, and are charged, the resulting electromagnetic field may be detectable [3]. Decaying currents may explain observed high-energy cosmic rays [3, 4], and could also provide a mechanism for baryogenesis [5, 6]. There are several processes which can create currents on strings. These include interaction with the plasma and collisions between cosmic strings. Charged currents can also be generated by magnetic or electric fields. Stable relics, vortons, can form as collapsing string loops are stabilized by the angular momentum of the trapped charge carriers [7]. If they form at high energy scales, these relics can dramatically alter the evolution of string networks. Vortons formed at low energies may provide a dark matter candidate [1].

As the universe expands, a network of cosmic strings will stretch, suggesting that its energy density will grow relative to everything else. This would lead to the universe becoming

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string-dominated, which is strongly ruled out by observation. More careful analysis shows that loops of string will break off from the network. These then contract, losing energy by gravitational radiation. This avoids string domination. If the loops are stabilized by conserved currents, this mechanism fails, allowing the corresponding theory to be severely constrained [8].

The cosmological implications of the vortons are most pronounced when the universe has become matter-dominated. If they decay during the era of radiation domination, the cosmological catastrophe may be avoided and the vorton bounds evaded. It has recently been realized that subsequent phase transitions can have a considerable effect on the microphysics of cosmic strings. Massless fermion currents on cosmic strings can be both created [9] and destroyed [10]. In this paper I examine the destruction of such currents in grand unified theories with an $SO(10)$ symmetry group.

I describe a simple cosmic string model in Section 2, and show that it has zero energy fermion solutions, called zero modes. This result was originally derived by Jackiw and Rossi [11]. I then show how the zero-mode solutions can be extended to massless currents.

In Section 3 I discuss the fate of zero modes on strings formed in an $SO(10)$ grand unified symmetry [10]. At high temperatures the theory resembles the toy model of Section 2. I investigate the implications of the electroweak phase transition for the zero modes, and show that they do not survive it.

I consider the implications of spectral flow in Section 4, and deduce that the currents acquire a small mass. The string current can now dissipate. This allows vortons to decay and weakens the cosmological bounds on such models [5]. An important feature that allows zero modes to be removed is the presence of a massless particle that mixes with the zero mode after the transition. Finally, I summarize the conclusions.

2 Fermion Zero Modes on an Abelian Cosmic String

The Abelian Higgs model has the Lagrangian

$$\mathcal{L} = (D_\mu \phi)^* (D^\mu \phi) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{\lambda}{4} (|\phi|^2 - \eta^2)^2 \tag{1}$$

with $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. As well as the usual $\phi = $ const. solution, it has cosmic string solutions of the form

$$\phi = \eta f(r) e^{in\theta} \tag{2}$$

$$A_\mu = n \frac{a(r)}{er} \delta_\mu^\theta \tag{3}$$

In order for the solution to be regular at the origin $f(0) = a(0) = 0$. If the solution is to have finite energy, $f(r)$ and $a(r)$ must tend to 1 as $r \to \infty$. The resulting string is the well known Nielsen-Olesen vortex [12]. It turns out that f and a take their asymptotic values everywhere outside of a small region around the string. Thus $|\phi|$ is constant and A_μ is pure gauge away from the string. The size of this region is of order η^{-1}.

Consider an extension of (1) to include a two-component fermion, with charge 1/2. The extra terms in the Lagrangian will then be

$$\mathcal{L}_{\text{fermions}} = \bar{\psi} i \sigma^\mu D_\mu \psi - \frac{1}{2} \left[i g_Y \bar{\psi} \phi \psi^c + (\text{h. c.}) \right] \tag{4}$$

where $\sigma^\mu = (-I, \sigma^i)$, $D_\mu \psi = \left(\partial_\mu - \frac{1}{2} i e A_\mu \right) \psi$, and $\psi^c = i \sigma^2 \psi^*$ is the charge conjugate of ψ. This gives the field equations

$$\left(\begin{array}{cc} -e^{i\theta} \left[\partial_r + \frac{i}{r} \partial_\theta + n \frac{a(r)}{2r} \right] & \partial_z + \partial_t \\ \partial_z - \partial_t & e^{-i\theta} \left[\partial_r - \frac{i}{r} \partial_\theta - n \frac{a(r)}{2r} \right] \end{array} \right) \psi - m_{\text{f}} f(r) e^{in\theta} \psi^* = 0 \tag{5}$$

where the expressions (2) and (3) have been substituted for ϕ and A_μ, and $m_{\text{f}} = g_Y \eta$.

Following previous work by Jackiw and Rossi [11] we look for solutions with no z or t dependence. Such solutions will have zero energy and are called zero modes. We can see from (5) that they can also be taken as eigenstates of σ^3. The angular dependence of a solution satisfying $\sigma^3 \psi = \psi$ can be separated out with the ansatz

$$\psi(r, \theta) = \binom{1}{0} \left(U(r) e^{il\theta} + V^*(r) e^{i(n-1-l)\theta} \right) \tag{6}$$

l is an arbitrary integer. We will consider the case $2l = n - 1$ separately. The field equations (5) imply

$$\left(\partial_r - \frac{l}{r} + \frac{na(r)}{2r} \right) U + m_{\text{f}} f V = 0$$
$$\left(\partial_r - \frac{n - 1 + l}{r} + \frac{na(r)}{2r} \right) V + m_{\text{f}} f U = 0 \tag{7}$$

Analytic solutions to these equations cannot generally be found. However it is possible to determine the number of physical solutions by considering their asymptotic form.

At large r we can approximate f and a by 1. Then (7) can be solved with modified Bessel functions, which are asymptotically equal to $e^{\pm m_{\text{f}} r} / \sqrt{r}$. We are interested in normalizable solutions, with $\int |\psi|^2 \, d^2x$ finite, so only the decaying solution is acceptable.

When r is small, we can neglect f and a. To first order the solutions of (7) are determined by the angular dependence of (6)

$$\begin{array}{ll} U \sim r^l, & V = O(r^{l+1}) \\ V \sim r^{n-1-l}, & U = O(r^{n-l}) \end{array} \tag{8}$$

In order to match up with the one acceptable large-r solution, both the small-r solutions must be regular. This will only be true if $0 \le l \le n - 1$. This suggests there are n solutions of (7) if $n > 0$ since there are n choices of l. In fact we have counted each solution twice, since putting $l \to n - 1 - l$ in (6) gives an equivalent ansatz. For every real solution of (7) there is also an imaginary one. This gives a total of n real solutions.

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The above analysis needs modification when $l = n - 1 - l$. We can set $V^* = U$ in (6), giving the single equation

$$\left(\partial_r - \frac{l}{r} + \frac{na(r)}{2r}\right) U + m_{\mathrm{f}} f U^* = 0 \tag{9}$$

This can be solved analytically. The solution which is well-behaved at large r is

$$\psi(r, \theta) = \binom{1}{0} r^l \exp\left(-\int_0^r m_{\mathrm{f}} f(s) + n\frac{a(s)}{2s} ds\right) \tag{10}$$

This is regular at $r = 0$ if $l = (n-1)/2 \geq 0$.

Similar analysis can be applied to the solutions satisfying $\sigma^3 \psi = -\psi$. We find a total of $|n|$ solutions, which are all eigenstates of σ^3. Their eigenvalues are $+1$ if $n > 0$, and -1 if $n < 0$. All the solutions decay exponentially outside the string, and so are confined to it. They can be regarded as fermions trapped on the string.

We can see from (5) that the solutions can easily be extended to include z and t dependence. This is achieved by multiplying ψ by $\alpha(z, t)$, which satisfies $(\partial_z \mp \partial_t)\alpha = 0$, depending on whether $\sigma^3 \psi = \pm \psi$. Thus the trapped fermions move at the speed of light in the $\pm z$ direction.

These currents are conserved, and the string acts as a perfect conductor. While zero modes have very little cosmological significance, the lightlike currents can have dramatic consequences.

3 An $SO(10)$ GUT with Strings

One example of a phenomenologically credible grand unified theory (GUT) has the symmetry breaking

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\begin{aligned}
SO(10) &\overset{\Phi_{126}}{\longrightarrow} SU(5) \times Z_2 \\
&\overset{\Phi_{45}}{\longrightarrow} SU(3)_c \times SU(2)_L \times U(1)_Y \times Z_2 \\
&\overset{\Phi_{10}}{\longrightarrow} SU(3)_c \times U(1)_Q \times Z_2
\end{aligned} \tag{11}
$$

The discrete Z_2 symmetry allows the formation of topologically stable cosmic strings. One possibility is an Abelian string, similar to that described in Section 2,

$$\Phi_{126} = e^{in\theta} \phi_0 f(r) \tag{12}$$

$$X_\theta = \frac{n}{\sqrt{10}} \frac{a(r)}{er} \tag{13}$$

ϕ_0 is the usual VEV of Φ_{126}, and X is a GUT gauge field broken by Φ_{126}. The only stable Abelian strings have $|n| = 1$, but higher winding number strings may have long lifetimes. Non-Abelian strings also form in this model, but they do not have zero modes at high temperatures, and I will not consider them here [10].

The string gauge field has a nontrivial effect on the electroweak Higgs field Φ_{10}. The components of Φ_{10} have charges $\pm 1/5$ with respect to the generator of the GUT gauge field X. If n is high enough, it is energetically favourable for Φ_{10} to wind like a string.

$$\Phi_{10} = H_u e^{im\theta} h_u(r) + H_d e^{-im\theta} h_d(r) \tag{14}$$

$$Z_\theta = \sqrt{\frac{5}{8}} \left(m - \frac{n}{5} \right) \frac{b(r)}{er} \tag{15}$$

where H_u and H_d are the usual VEVs of the components of Φ_{10}. Whether Φ_{10} winds or not, a nonzero electroweak gauge field is required to give a vanishing covariant derivative outside the string. The solution satisfies the boundary conditions $h_u(\infty) = h_d(\infty) = b(\infty) = 1$, $b(0) = 0$ and $h_u(0) = h_d(0) = 0$ if $m \neq 0$. If $m = 0$ h_u and h_d are small, of order $|H_{u,d}|/|\phi_0|$ [10]. m is determined by the GUT string, and is equal to the nearest integer to $n/5$, so the electroweak Higgs field does not wind around a topologically stable Abelian string. The region of electroweak symmetry restoration is inversely proportional to the electroweak scale, so is far greater than the string core.

The fermion sector of the $SO(10)$ GUT contains all the Standard Model fermions, and an extra right-handed neutrino for each family. For simplicity I will just consider the string's effect on just one family, although it is easy to generalize the results. Of the three Higgs fields, only Φ_{126} and Φ_{10} can couple to fermions. Only right-handed neutrinos couple to ϕ_0, while neutrinos of either helicity couple to H_u. As the gauge symmetry unifies left- and right-handed particles, it is convenient to express everything in terms of left-handed spinors. I will use $\nu = \nu_L$ and $\nu^c = i\sigma^2 \bar\nu_R^T$ to express neutrino terms. Defining $\psi^{(\nu)} = (\nu^c, \nu)^T$, the resulting neutrino mass terms are

$$\bar\psi^{(\nu)} \begin{pmatrix} m_{\mathrm{GUT}} f(r) e^{in\theta} & m_{\mathrm{u}} h_u(r) e^{im\theta} \\ m_{\mathrm{u}} h_u(r) e^{im\theta} & 0 \end{pmatrix} \psi^{(\nu)} \tag{16}$$

$m_{\mathrm{u}} \sim |H_u| \sim 1$ MeV is the up-quark mass and $m_{\mathrm{GUT}} \sim |\phi_0| \sim 10^{16}$ GeV. Since $\epsilon = m_{\mathrm{u}}/m_{\mathrm{GUT}} \ll 1$, the neutrino mass eigenvalues outside the string are

$$m_R = m_{\mathrm{GUT}} \frac{\sqrt{1 + 4\epsilon^2} + 1}{2} \approx m_{\mathrm{GUT}}$$

$$m_L = m_{\mathrm{GUT}} \frac{\sqrt{1 + 4\epsilon^2} - 1}{2} \approx \frac{m_{\mathrm{u}}^2}{m_{\mathrm{GUT}}} \tag{17}$$

The mass eigenstates are then approximately $\nu^c + \epsilon\nu$ and $\nu - \epsilon\nu^c$. This illustrates the seesaw mechanism [13]. Although the neutrinos have the same couplings to the electroweak Higgs field as the up quark, the GUT Higgs ensures that ν_R is superheavy and ν_L is very light, as is required to agree with observation. Recent measurements have suggested that ν_L does indeed have a small mass [14].

At high temperatures only Φ_{126} is nonzero and $m_{\mathrm{u}} = 0$. The model then reduces to the toy model discussed in Section 2, with $\psi = \nu^c$. Thus $|n|$ right-handed neutrino zero modes exist on the string. This implies that such strings always have conserved currents at high temperatures. These could allow vortons to be formed. If string loops formed at this

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energy scale do not decay, their evolution will lead to serious conflict with observations of the present universe [8].

The situation is more complex for the neutrino fields at low temperatures since they couple to two Higgs fields at the same time. As in Section 2, we will first look for solutions satisfying $\sigma^3\psi = \psi$. With respect to the generator of the X gauge field, ν has charge 3/10. It has the same charge as Φ_{10} with respect to the Z generator. The resulting field equations are

$$e^{i\theta}\left(\partial_r + \frac{i}{r}\partial_\theta + \frac{n}{2}\frac{a(r)}{r}\right)\nu^c + m_u h_u(r)e^{im\theta}\nu^* + m_{\text{GUT}}f(r)e^{in\theta}\nu^{c*} = 0 \qquad (18)$$

$$e^{i\theta}\left(\partial_r + \frac{i}{r}\partial_\theta - \frac{3n}{10}\frac{a(r)}{r} + \left[m - \frac{n}{5}\right]\frac{b(r)}{r}\right)\nu + m_u h_u(r)e^{im\theta}\nu^{c*} = 0 \qquad (19)$$

Although Jackiw and Rossi did not consider this case, it can be approached using a similar method to theirs. The angular dependence can be removed with the substitutions

$$\nu^c = \begin{pmatrix}1\\0\end{pmatrix}\left(Ue^{il\theta} + V^*e^{i(n-1-l)\theta}\right) \qquad (20)$$

$$\nu = \begin{pmatrix}1\\0\end{pmatrix}\left(W^*e^{-i(m+1+l)\theta} + Ye^{-i(m+n-l)\theta}\right) \qquad (21)$$

(The case $2l = n - 1$ will be considered later.) The resulting four complex differential equations are

$$\left(\partial_r - \frac{l}{r} + \frac{na(r)}{2r}\right)U + m_u h_u(r)W + m_{\text{GUT}}f(r)V = 0 \qquad (22)$$

$$\left(\partial_r - \frac{n-1-l}{r} + \frac{na(r)}{2r}\right)V + m_u h_u(r)Y + m_{\text{GUT}}f(r)U = 0 \qquad (23)$$

$$\left(\partial_r + \frac{m+1+l}{r} + \frac{(10m-2n)b(r) - 3na(r)}{10r}\right)W + m_u h_u(r)U = 0 \qquad (24)$$

$$\left(\partial_r + \frac{m+n-l}{r} + \frac{(10m-2n)b(r) - 3na(r)}{10r}\right)Y + m_u h_u(r)V = 0 \qquad (25)$$

Splitting these equations into real and imaginary parts gives two identical sets of four real equations, and so it is only necessary to look for real solutions.

When r is large, h_u, f, a, and b are all approximately 1. As with (7), rearrangement of (22)–(25) reduces them to modified Bessel equations at large r. The four independent solutions are proportional to $e^{\pm m_R r}/\sqrt{r}$ and $e^{\pm m_L r}/\sqrt{r}$, so only two of them are normalizable.

For small r, $h_u \sim r^{|m|}$, $f \sim r^{|n|}$, and the gauge terms are of order r. Thus we can ignore them when finding the leading-order terms of the small-r solutions of (22)–(25). The dominant terms of the four independent solutions are

$$r^l, \quad r^{n-1-l}, \quad r^{-m-1-l}, \quad r^{l-n-m} \qquad (26)$$

If φ is a solution of (22)–(25) for all r, which is normalisable at $r = \infty$, then it will have to match some combination of the two normalizable solutions for large r. At $r = 0$, φ will be made up of a combination of the solutions in (26). So if φ is to be normalizable everywhere, at least three of the solutions (26) must be well behaved at $r = 0$. Thus for

each l satisfying three of the inequalities $l \geq 0$, $l \leq -m - 1$, $l \leq n - 1$ and $l \geq n + m$, there will be one normalizable solution. If l satisfies all four there will be two solutions. Not all of these solutions are independent since the real (or imaginary) solutions for $l = l'$ and $l = n - 1 - l'$ are proportional.

For $l = (n - 1)/2$ the angular dependence of (18) and (19) is removed with the substitutions

$$\nu^c = U e^{il\theta} , \qquad \nu = W^* e^{-i(m+1+l)\theta} \tag{27}$$

giving (after dropping gauge terms) the equations

$$\left(\partial_r - \frac{l}{r} + \frac{na(r)}{2r} \right) U + m_{\mathrm{u}} h_u(r) W + m_{\mathrm{GUT}} f(r) U^* = 0 \tag{28}$$

$$\left(\partial_r + \frac{m + 1 + l}{r} + \frac{(10m - 2n)b(r) - 3na(r)}{10r} \right) W + m_{\mathrm{u}} h_u(r) U = 0 \tag{29}$$

The two real solutions have the asymptotic forms $e^{-m_R r}/\sqrt{r}$ and $e^{m_L r}/\sqrt{r}$, while the two imaginary solutions are $e^{m_R r}/\sqrt{r}$ and $e^{-m_L r}/\sqrt{r}$. The leading-order terms of the small-r solutions (real or imaginary) are

$$r^l , \quad r^{-m-1-l} \tag{30}$$

Matching large- and small-r solutions reveals that in this case there is one real and one imaginary solution if $0 \leq l \leq -m - 1$.

Combining all the above results gives a grand total of $2m$ (m real and m imaginary) normalizable solutions if $m > 0$, and 0 otherwise. Surprisingly, this does not depend on n. A similar approach can be applied to the other components of ν^c and ν to give $-2m$ normalizable solutions, provided $m < 0$. Hence there are $2|m|$ possible neutrino zero modes after electroweak symmetry breaking. Examination of the asymptotic zero-mode solutions reveals that they are confined to the region of electroweak symmetry restoration.

Since $|2m| < |n|$, some of the zero modes will be destroyed. For a stable $n = 1$ string all zero modes are destroyed. Thus, since higher n strings almost certainly decay, there are zero modes before, but not after the electroweak phase transition. The neutral current in the string disperses [15] and any vortons formed would dissipate after about 10^{-10} sec [5]. Before the electroweak phase transition from about 10^{10} GeV to 10^2 GeV the universe would undergo a period of matter domination. Once the vortons dissipate there would be some reheating of the universe. However, the electroweak interactions and physics below the phase transition would be unaffected.

Although we have looked at a specific $SO(10)$ symmetry breaking, the results apply to most other breakings of this group, since they have the same fermion mass terms. In a more arbitrary $U(1) \times$(Standard Model) theory, the ratio of m and n could be greater than 1, in which case extra zero modes would be created at the electroweak phase transition.

Although we have only considered one type of model, the arguments can be applied to any theory with fermions coupling to cosmic strings. This has been done for a general theory in ref. [16]. The results obtained agree with simpler index theorems derived previously [17].
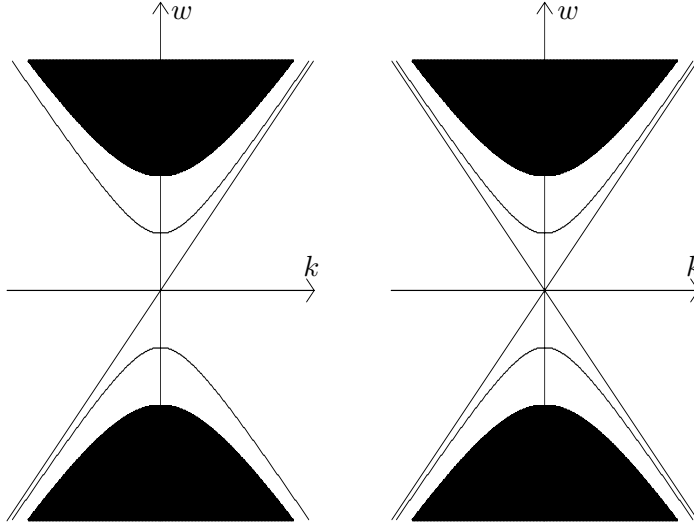
Figure 1: The Dirac spectrum with a zero mode (left) and a very low lying bound state (right). Both spectra also have a bound state and continuum.

4 Index Theorems and Spectral Flow

I have shown that zero modes can acquire masses at subsequent phase transitions. No matter how small this mass, the spectrum of the Dirac operator changes significantly. If we compare the Dirac spectrum with a zero mode and a low-lying bound state with infinitesimal mass (Fig. 1), we see that an arbitrarily small perturbation to the zero mode introduces an entire new branch to the spectrum. Any massive state gives a spectrum that is symmetric about both the w and k axes; there is always a reference frame in which the particle is at rest and others where it is moving up or down the string. Conversely, the zero mode, which is massless, can only move in one direction along the string and its spectrum is asymmetric. The transition from zero mode to low-lying bound state causes drastic changes in the spectrum and can be brought about by infinitesimal changes in the value of one Higgs field. If we consider the species with the zero mode alone, this infinite susceptibility to the background fields appears unphysical. However, when we include the massless neutrino in the $SO(10)$ model the spectral changes are less worrying. For a small coupling between the two neutrinos, both the before and after spectra have a continuum of massless or nearly massless states. These states can be used to build the extra branch of the perturbed zero-mode spectrum, allowing small changes in the overall spectrum for small changes in the background fields. This observation leads me to conjecture that zero modes can be removed only if they become mixed with other massless states.

At the electroweak phase transition the neutrino zero mode will mix with the left-handed neutrino field to form a bound state. Its mass will be proportional to the neutrino mass inside the string, which is of order $m_{\mathrm{u}}^3/m_{\mathrm{GUT}}^2$ [18]. High energy currents are free to scatter into left-handed neutrinos off the string, so the maximal current will be very small. This will be insufficient to stabilize vortons. Additionally, the current will be spread over the

region of electroweak symmetry restoration. This is far larger than the size of a vorton, which is a couple of orders of magnitude greater than the GUT string radius [7]. The current on one part of the string will then interact with current on the opposite side of the loop, increasing the vorton's instability. Thus vortons in $SO(10)$ will certainly decay at the electroweak phase transition.

Although I have discussed a specific type of GUT, many of the ideas I have used apply to a far wider range of theories. Fermion zero modes are generic in supersymmetric theories [19]. They, too, can be destroyed at lower temperatures, in this case by supersymmetry breaking [20].

5   Conclusions

If fermions couple to a cosmic string Higgs field, their spectrum will gain extra states which are confined to the string core. The existence of zero-energy fermion states on strings can be examined analytically. Such solutions can easily be extended to give massless currents, which will have significant cosmological effects. In this paper I have investigated the existence of fermion zero modes in two simple models, one of which is contained in a realistic $SO(10)$ GUT. We have seen that the microphysics of cosmic strings can be influenced by subsequent phase transitions. Fermion zero modes, and consequently lightlike currents on the strings can be created or destroyed by such phase transitions. In determining whether or not a cosmic string carries conserved currents it is not enough to just consider them at formation, but one must follow the microphysics through the multiple phase transitions that the system undergoes.

It is possible for vortons formed at high energy to dissipate after a subsequent phase transition if the relevant fermion zero mode does not survive the phase transition, thus vorton bounds could be evaded. I have demonstrated this effect by analysing the neutrino zero modes of an $SO(10)$ model in detail. Prior to dissipation there could be a period of vorton domination; after the phase transition the universe would reheat and then evolve as normal.

The right-handed neutrino zero modes are removed at the electroweak phase transition when they mix with the Standard Model left-handed neutrinos. By considering spectral flow we have seen that zero modes and massless currents can only be removed by mixing them with another massless field. The resulting state will be a bound state or massive current. In the model I considered, the maximal current that the string can support after the electroweak phase transition is proportional to the left-handed neutrino mass. Such currents are far too small to stabilize vortons.

Acknowledgements

on cosmology for providing such an enjoyable and interesting conference, and for inviting me.

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